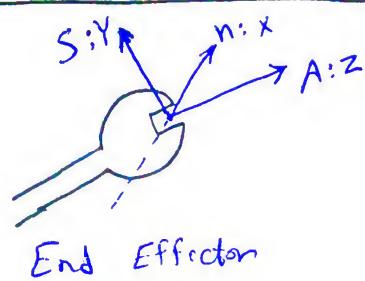


# Robotics

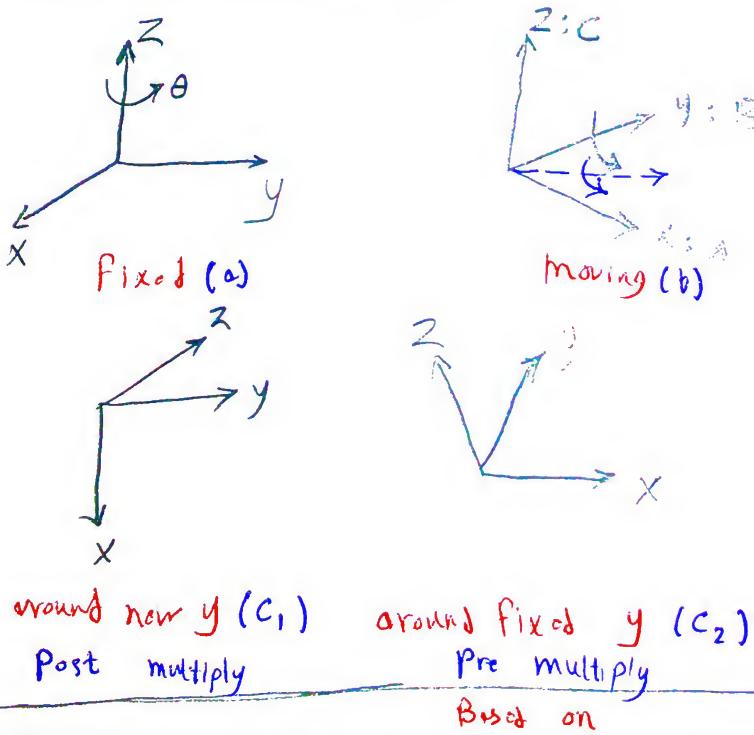
## Section 4

### Rotation Matrix

- 1- Orientation of Frame with another
- 2- Coordinates of Point  $\underline{P}$  with different frame
- 3- Rotation Operator



### Composition and Rotation



### Successive Rotation (Similarity Transformation)

$$R_{C_1} = R(z, \theta) \underset{\substack{\text{Fixed} \\ \text{Pre multiply}}}{\mathcal{I}} R(y, \phi) \underset{\substack{\text{Current} \\ \text{Post multiply}}}{\mathcal{I}} R(z, \alpha)$$

$$R_{C_2} = R(y, \phi) R(z, \alpha) \mathcal{I}$$

### Example

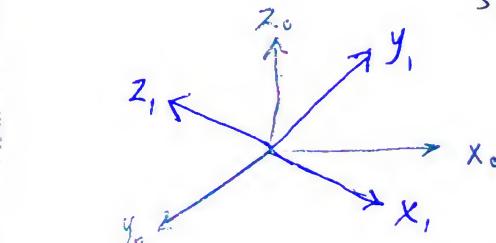
$R$ : Specified by the following sequence

- 1- rotation of  $\theta$  about current  $X$ -axis
- 2- rotation of  $\phi$  about current  $Z$ -axis
- 3- rotation of  $\alpha$  about fixed  $Z$ -axis
- 4- rotation of  $\beta$  about current  $Y$ -axis
- 5- rotation of  $\delta$  about fixed  $X$ -axis

$$R = R(x, \delta) R(z, \alpha) \mathcal{I} R(x, \theta) R(z, \phi) R(y, \beta)$$

### Parametrization of Rotations

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3 \times 3}$$



الベクトル  $\underline{y}_1$  は  $\underline{x}_0$  と  $\underline{z}_0$  の和である

Euler Angles  $(ZYZ)$   $(ZBC)$   $(\alpha\phi\psi)$  Representation I

$$R = R(z, \theta) R(y, \phi) R(z, \psi)$$

$$= \begin{bmatrix} C\phi C_\theta C_\psi - S_\theta S_\psi & C_\phi S_\theta C_\psi - S_\phi S_\psi & C_\phi S_\psi \\ S\phi C_\theta C_\psi + C_\theta S_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & S_\phi S_\psi \\ -S\phi C_\psi & C_\phi S_\psi & C_\phi \end{bmatrix}$$

## Euler Angles (Z X Z) (Z A C) Representation II

$$R = R(z, \theta) R(x, \phi) R(z, \psi)$$

Roll - Pitch - Yaw  
z      y      x

الدور (z) والرُّفع (y) والدور (x)

\*Fixed elementary axis

- 1-  $\psi$  around x
- 2-  $\phi$  around y
- 3-  $\theta$  around z

$$R = R(z, \theta) R(y, \phi) R(x, \psi)$$